THE INSTANTON DENSITY AT FINITE TEMPERATURES

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Abstract

For *low* T new strict results for the instanton density n(T) are reported. Using the PCAC methods, we express n(T) in terms of *vacuum* average values of certain operators, times their *calculated* T-dependence. At high T, we discuss the *applicability* limits of the perturbative results. We further speculate about possible behaviour of n(T) at $T \sim T_c$.

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1 Introduction

Tunneling between topologically different configurations of the gauge field, described semiclassically by instantons [1], dominate the physics of light quarks. In early works (summarized e.g. in [2]) instantons were treated as a dilute gas, while later it was recognised that the instanton ensemble resemble rather a strongly interacting "instanton liquid" [5, 6]. During the last years calculations of the correlation functions [7] and Bethe-Salpeter wave functions [8] for various mesonic and baryonic channels were made along these lines. The results agree surprisingly well both with phenomenology [9] and lattice simulations [10]. Parameters of the "instanton liquid" were also reproduced (by the "cooling" method) directly from the (quenched) lattice configurations [11]. In addition to that, it was found that correlation functions as well as hadronic wave functions in most channels remain practically unchanged after "cooling". In particular, main mesons and e.g. the nucleon remains bound, with about the same mass and wave function. This confirms that the agreement of previous lattice calculations with the instanton model was not accidental, and instantons indeed are the most important non-perturbative phenomena in QCD.

Investigations of the finite temperature case were started in [12], where classical *caloron* solution was found. Although the solution depends on T, the action is T-independent. Furthermore, it was argued by one of us [14] that at high T the specific charge renormalization and the Debye-type screening of the electric field in quark-gluon plasma should suppress instantons with size $\rho > 1/T$. Pisarski and Yaffe [22] have evaluated the T-dependence in the one-loop approximation. The physical nature of their result and its applicability region will be discussed below.

Last years the studies of the finite temperature had focused especially on the region around the chiral phase transition $T \approx T_c$. The first attempt to understand this phase transition as a rearrangement of the instanton liquid, going from a random phase at low temperatures to a strongly correlated "molecular" phase at high

temperatures was made in [13]. Recently this idea was recently made more quantitative in [15, 16], and although the detailed comparison to lattice thermodynamics and correlation functions is yet to be made, the first results show overall agreement, indicating that the mechanism of chiral restoration is basically understood.

The particular topic of this paper, the *temperature dependence of the instanton density* n(T), is certainly an important ingredient of all this development. However, it has attracted surprisingly little attention in literature. The only attempts to determine this quantity by "cooling" of the lattice configurations (same method as was used at T=0) were made in refs.[17]. The only statement one can probably make using these data is that the density has *no* significant T-dependence, till $T \sim T_c$. Unfortunately, accuracy of this statement remains at the level of 50%, at best.

2   Instantons at low temperatures

As far as we know, the modification of the instanton density in this limit was never considered before. However, the general physical picture at low $T << T_c$ is well known: the heat bath is just a dilute gas of the lightest hadrons, the pions. The problem is especially clear in the chiral limit, in which quark and pion masses are neglected, and this case is assumed in what follows. Due to their Goldstone nature, the large wavelength pions are nothing else but small collective distortions of the quark condensate. Therefore, one can always translate the average values over the *pion state* as the *vacuum expectation value* (VEV) of a different but related quantity.

Let us now consider a tunneling event, described semiclassically by the instanton solution. As discovered by 't Hooft [3], it can only take place if certain rearrangements in the fermionic sector are made, which can be described by some effective Lagrangian with $2N_f$ fermionic legs. For simplicity, in this work we assume that the number of light flavors $N_f = 2$, disregarding strange and heavier quarks.

The situation can be further simplified by consideration of *small-size* instantons

$\rho\Lambda_{QCD} << 1$, for which this Lagrangian can be considered as a *local* operator. What follows from 't Hooft Lagrangian, after averaging over the instanton orientations is made, is [18]:

$$\Delta\mathcal{L} = \int d\rho d_0(\rho)(\frac{4}{3}\pi^2\rho^3)^2\{\bar{q}\Gamma_+q\bar{q}\Gamma_-q + \frac{3}{32}\bar{q}\Gamma_+^a q\bar{q}\Gamma_-^a q - \frac{9}{128}\bar{q}\Gamma_+^{a\mu\nu}q\bar{q}\Gamma_-^{a\mu\nu}q\} \quad (1)$$

where the definition of the operators involved is as follows

$$\Gamma_\pm = (\frac{1-\gamma_5}{2}) \otimes (\frac{1\pm\tau_3}{2}) \quad (2)$$

$$\Gamma_\pm^a = (\frac{1-\gamma_5}{2}) \otimes (\frac{1\pm\tau_3}{2}) \otimes t^a \quad (3)$$

$$\Gamma_\pm^{a\mu\nu} = (\frac{1-\gamma_5}{2})\sigma_{\mu\nu} \otimes (\frac{1\pm\tau_3}{2}) \otimes t^a \quad (4)$$

At T=0 the instanton density is therefore proportional to the VEV of $\Delta\mathcal{L}$, and the only thing which changes at low T is clearly the modification of the quantities above.

How to do this technically was actually clarified by PCAC-related paper in 60's. The necessary formulae can be found e.g. in the recent paper by Eletsky [19], where different set of four-fermion operators (appearing in QCD sum rules for vector and axial currents) was considered. The general expression is

$$\langle(\bar{q}Aq)(\bar{q}Bq)\rangle_T = \langle(\bar{q}Aq)(\bar{q}Bq)\rangle_0 - \frac{T^2}{96F_\pi^2}\langle(\bar{q}\{\Gamma_5^a\{\Gamma_5^a A\}\}q)(\bar{q}Bq)\rangle_0$$

$$- \frac{T^2}{96F_\pi^2}\langle(\bar{q}Aq)(\bar{q}\{\Gamma_5^a\{\Gamma_5^a B\}\}q)\rangle_0 - \frac{T^2}{48F_\pi^2}\langle(\bar{q}\{\Gamma_5^a A\}q)(\bar{q}\{\Gamma_5^a B\}q)\rangle_0 \quad (5)$$

where A,B are arbitrary flavor-spin-color matrices and $\Gamma_5^a = \tau^a\gamma_5$. Here and below flavor matrices are shown as τ_a, and color one as t^a.

For generality, there are six different operators of different spin-flavor-color structure involved (see the Table). Their T-dependence in $O(T^2)$ order can be found

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from the expression above, and it is also listed in the Table. Generally, the operators mix, and it is convenient to group those combinations which do not. Returning to the effective Lagrangian, one can see that there are only two combinations which are actually relevant

$$K_1 = \mathcal{O}_1^A + \frac{3}{32}\mathcal{O}_1^B - \frac{9}{128}\mathcal{O}_1^C \tag{6}$$

$$K_2 = \mathcal{O}_2^A + \frac{3}{32}\mathcal{O}_2^B - \frac{9}{128}\mathcal{O}_2^C \tag{7}$$

The final result for the instanton density at low temperature T therefore contains two constants, the *vacuum* averages of these operators

$$dn(T) = \frac{d\rho}{\rho^5}d(\rho)(\frac{4}{3}\pi^2\rho^3)^2 \left[\langle K_1 \rangle_0 \frac{1}{4}(1 - \frac{T^2}{6F_\pi^2}) - \langle K_2 \rangle_0 \frac{1}{12}(1 + \frac{T^2}{6F_\pi^2}) \right] \tag{8}$$

Although two VEV's which appear here are unknown (and are subject to further investigations), it is clear that the total T-dependence should be *rather weak*: it is bound to be $1 + a\frac{T^2}{F_\pi^2}$ with a in the strip $a = (-1/6, 1/6)$.

The so called *vacuum dominance* (VD) hypothesis [18] was used in various applications (such as QCD sum rules and weak decays) for evaluation of VEV's of various operators. It leads to VEV's and the $O(T^2)$ corrections also indicated in the Table 1. Remarkably enough [1], in this case the T-dependence *exactly cancel*.

Returning to discussion of our general result (8), we comment that this result by itself rules out some possible picture of low-T vacuum structure. In particular, the so called "random instanton liquid model" (RILM) was shown to be a reasonable approximation for the T=0 case [7]. One may wander if the same model can describe the T-dependence at low T. If it is the case, the quark condensate should scale with the instanton density as $n_{inst}^{1/2}$, see [5]. Being combined with the well known chiral

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[1] (However, the reader should be warned that in general this approximation is not supposed to hold or be very accurate, particularly for the operators considered, which are related with instantons.

theory result

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 (1 - \frac{T^2}{8F_\pi^2})$$ (9)

these two formulae lead to[2] $n_{inst}(T)/n_{inst}(0) = (1 - \frac{T^2}{4F_\pi^2})$.

However, as this estimate happen to be *outside* of the strip indicated above, this possibility is definitely *ruled out*. It means that, even if RILM is a perfect model at T=0, it cannot be so for even small T. This conclusion agrees very well with other studies of the instanton ensemble, such as [15], which emphasize the role of correlations built up with growing T in the ensemble of instantons.

3   Instantons at high temperatures

QCD vacuum at high temperatures undergoes a phase transition into a new phase, called the *quark-gluon plasma*[20]. Although *virtual* gluons *antiscreen* the external charge (the asymptotic freedom), the *real* gluons of the perturbative heat bath *screen* it, leading to the well known expression for the Debye screening mass [20]:

$$M_D^2 = (N_c/3 + N_f/6)g^2T^2$$ (10)

where N_c, N_f are the numbers of colors and flavors, respectively. "Normal" O(1) electric fields are therefore screened at distances $1/gT$, while stronger "non-perturbative" fields of the instantons $O(1/g)$ should be screened already at scale 1/T.

Quantitative behaviour of the instanton density at high temperatures was determined in ref. [22].

$$dn(\rho, T) = dn(\rho, T = 0) \exp\left\{ -\frac{1}{3}\lambda^2(2N_c + N_f) - 12A(\lambda)\left[1 + \frac{1}{6}(N_c - N_f)\right]\right\}$$ (11)

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[2] Note that the same result can be obtained by a naive assumption, which was used in some works on QCD sum rules in the past: namely, that average of *all* four-fermion operators have T-dependence as the *square* of the condensates.

where $\lambda = \pi\rho T$, and $A(\lambda) = -\frac{1}{36}\lambda^2 + o(\lambda^2)$. Therefore, at high temperatures the contribution of small size instantons such as $T >> 1/\rho$ is exponentially suppressed. As a result, the instanton-induced contribution to physical quantities like energy density (or pressure, etc) become of the order of[3]

$$\epsilon(T) \sim \int_0^{1/T} \frac{d\rho}{\rho^5} (\rho\Lambda)^{(11N_c/3)} \sim T^4 (\Lambda/T)^{(11N_c/3)} \qquad (12)$$

which is small compared to that of ideal gas $\epsilon(T)_{ideal} \sim T^4$.

Although the Pisarski-Yaffe formula contains only the dimensionless parameter λ, its applicability is limited by *two separate* conditions:

$$\rho << 1/\Lambda, \quad T >> \Lambda \qquad (13)$$

The former condition ensure semiclassical treatment of the tunneling, while the latter is needed to justify perturbative treatment of the heat bath. In this section we would like to discuss applicability conditions of these well-known results in greater details.

Our first point is that the one-loop effective action discussed by Pisarski and Yaffe actually consists of *two parts* with very different physical origin and interpretation. To show that in the simplest case, consider the determinant corresponding to scalar isospin $1/2$ field[4] and rewrite them as follows:

$$\delta = Tr_T[\log(\frac{-D^2(A(\rho,T))}{-\partial^2})] - Tr[\log(\frac{-D^2(A(\rho))}{-\partial^2})] = \delta_1 + \delta_2 \qquad (14)$$

$$\delta_1 = Tr_T[\log(\frac{-D^2(A(\rho))}{-\partial^2})] - Tr[\log(\frac{-D^2(A(\rho))}{-\partial^2})] \qquad (15)$$

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[3]Here we consider only the pure glue theory. In the theory with massless fermions individual instantons are impossible, and only "instanton-antiinstanton" molecules can appear at high temperatures.

[4]The determinants of the actual quadratic fluctuations of the *quark* and *gluon* fields (modulo the factor corresponding to zero fermion modes) can be expressed via the determinants of *scalar* fields with isospins $1/2$ and 1 [22].

$$\delta_2 = Tr_T[\log(\frac{-D^2(A(\rho,T))}{-\partial^2})] - Tr_T[\log(\frac{-D^2(A(\rho))}{-\partial^2})] \tag{16}$$

Here Tr_T is a trace over all matrix structures, plus integration over $\mathcal{M}$ – the strip in R^4 with span in the τ direction of $1/T$. $A(\rho,T)$ is the caloron field and $A(\rho)$ is the instanton field. Two contributions introduced in this way, δ_1, δ_2, are the origin of two terms in the resulting formula (11).

As it was shown in ref.[21], the first term can be expressed via the *forward scattering amplitude* of heath bath constituents, on the instanton field. Therefore its physical origin is clear: ρ^2 comes from the scattering amplitude, while the temperature factor T^2 enters via the standard thermal integral over the particle momenta:

$$\delta_1 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p(\exp(p/T)-1)} TrT(p,p) \tag{17}$$

Let us show how it works using the example of a "scalar quark", which is simpler than realistic spinor and vector particles considered in [21]. One can evaluate $T(p,p)$, the forward scattering amplitude of a scalar quark on the instanton field, using standard Leman-Simansik-Zimmermann reduction formula:

$$TrT(p,p) = \int d^4x d^4y \ e^{ip.(x-y)} Tr(\partial_x^2 \Delta_{\frac{1}{2}}(x,y)\partial_y^2) \tag{18}$$

where $\Delta_{\frac{1}{2}}$ is the (isospin 1/2) scalar quark propagator [24]:

$$\Delta_{\frac{1}{2}}(x,y) = \frac{x^2y^2 + \rho^2 x.\tau y.\tau^\dagger}{4\pi^2(x-y)^2 x^2 y^2(1+\rho^2/x^2)^{\frac{1}{2}}(1+\rho^2/y^2)^{\frac{1}{2}}} \tag{19}$$

By rescaling (18) as $\xi = px, \eta = py$, subtracting the trace of the free propagator and going to the physical pole $p^2 = 0$, one gets:

$$TrT(p,p) = \int d^4\xi d^4\eta e^{in.(x-y)} \frac{\rho^2}{2\pi^2(\xi-\eta)^2} \left(\frac{\xi.\eta}{\xi^2\eta^2} - \frac{1}{2\xi^2} - \frac{1}{2\eta^2} \right) = -4\pi^2\rho^2 \tag{20}$$

As it is just constant, there is no problem with its analytic continuation to small Minkowski momenta of scattered quarks, and plugging (20) into (17) we have:

$$\delta_1 = \frac{1}{3}\eta\lambda^2, \ \eta = \left\{ \begin{array}{ccc} 1 & for & periodic \quad fields \\ -1/2 & for & antiperiodic \quad fields \end{array} \right\} \tag{21}$$

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Note also, that this scattering amplitude has the same origin (and the same dependence on N_c, N_f) as the Debye mass.

Although formally any result obtained by the perturbative expansion demand smallness of the effective charge $g(T) << 1$, it is not clear in practice what this condition actually imply. However, we conjecture that accuracy of calculation sketched above is controlled by the same effects as the accuracy of *perturbative calculation of the Debye mass* by itself. If so, one can use available lattice studies of the screening phenomena (e.g.[25]) and check at which T their results start to agree with the perturbative formula (10). We then conclude, based on available lattice data, that Debye mass and instanton suppression formula (11) should be valid above $T > T_{pert} = 3T_c \approx 500$ MeV).

How strong can this suppression be, at that point? Using a canonical "instanton liquid" size of the instanton $\rho \approx 1/3$ fm, one gets suppression on the level 10^{-3}, from the δ_1 term alone. It suggests a very dramatic behaviour in the interval from 1 to 3 T_c.

Let us further speculate about the magnitude of δ_1 contribution for lower temperatures. In the interval between T_c and T_{pert} it is expected on general ground (and observe on the lattice) that the Debye mass $M_D \to 0$ at the critical temperature: screening is gone together with the plasma. However, another suppression mechanism should substitute it *below T_c*, namely the one due to scattering of *hadrons* on the instanton. This is what we have done above for the low-T case, in which only the soft pions should be included. At this time, we do not know how to estimate this effect including other hadrons.

Now we turn to discussion of the second term δ_2 in (14), $A(\lambda)$ in [22], which was actually first obtained by Brown and Creamer in [23]. At small λ it leads to the following correction

$$\delta_2 = -(1/36)\lambda^2 + o(\lambda^2) \tag{22}$$

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and thus it has the same sign as δ_1 and parametric magnitude, just numerically smaller coefficient. (In the isospin 1 case $\delta_1 = (4/3)\lambda^2$ and $\delta_2 = -(4/9)\lambda^2 + o(\lambda^2)$.)

The splitting of the variation of the effective action into two physically different contributions is a generic phenomenon. This second term has different physical origin, because it is connected to a *quantum correction* to the colored current, times the *T-dependent variation* of the instanton field, the difference between the caloron and the instanton.

Thus, the finite T effects not only lead to appearance of a usual (perturbative) heat bath, but they also modify strong ($O(1/g)$) *classical gauge field* of the instanton. In Matsubara formalism this is described by the non-linear "interference" of the instanton field with its "mirror images", in the (imaginary) time direction.

Let us conjecture, that for $T < T_c$ this suppression mechanism is actually irrelevant, for the following reason. It is well known that in this T domain all gluonic correlators decay strongly with distance, because all physical "glueballs" states are very heavy. It should make any interaction with the "mirror images" (at distance $\beta = 1/T$) virtually impossible. Estimating this effect as $\exp(-M_{glueball}/T)$, where $M_{glueball} \sim 1.6 GeV$ is the mass of the lightest glueball, one gets even at $T = T_c \sim 140 MeV$ a suppression factor of the order of 10^{-5}.

4   Summary and discussion

We have studied the change of the instanton density at *low* and *high* temperatures.

In the former case, $T << T_c$, we have considered the heat bath as being made of dilute soft pions. Applying PCAC methods (in the chiral limit) we have derived strict result (8) for the instanton density n(T) at low T. It implies very weak T dependence, which agrees with available lattice measurements inside their (so far rather poor) accuracy. It also contradicts to some naive models, for example it shows that the "random instanton liquid model", presumably a good description of

the QCD vacuum, can not be true even at low T. Fortunately, it perfectly agrees with the current ideas about the finite-T QCD [15], pointing out that quark-induced instanton- antiinstanton correlations are building up with T, till only instanton - antiinstanton "molecules" remain for $T > T_c$.

Our discussion of the *high* temperatures can be summarized as follows. For very high $T > T_{pert} \sim 3T_c$ the perturbative result of Pisarski and Yaffe [22] holds.

Furthermore, we have pointed out that it consists of two parts, δ_1, δ_2, with different underlying physics. The first one is directly connected to occupation densities of quarks and gluons from the plasma. It is the same effect as lead e.g. to the Debye screening mass, so one knows from lattice data at which T this part of the instanton suppression can be trusted. We also claimed that it weakens toward T_c, but at the same time scattering of hadrons on instantons should appear at $T < T_c$, and we do not know how to take it into account (except for soft pions).

The second term δ_2 originates from the *T-dependent variation of the classical field*, coupled to a quantum correction to the colored current. We expect this effect to become exponentially small for $T < T_c$, but we do not know its T-dependence in the strip 1-3 T_c, where it can be very strong.

Finally, let us repeat once more, that understanding of the temperature dependence of the instanton density is of crucial importance for understanding of non-perturbative phenomena at and around T_c. Surprisingly little efforts has been made to clarify this question. In particularly, we call upon lattice community to make quantitative measurements of n(T), which can be done by well known methods.

5 Table

Operator	Coeff. in $\mathcal{L}$	T-renormalization	Vacuum Dominance Hypothesis
$\mathcal{O}_1^A =$ $\bar{q}\frac{1-\gamma_5}{2}q\bar{q}\frac{1-\gamma_5}{2}q$	$\frac{1}{4}$	$\langle\mathcal{O}_1^A\rangle_0 - (3\langle\mathcal{O}_1^A\rangle_0 + \langle\mathcal{O}_2^A\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(132 - 360\frac{T^2}{12F_\pi^2})$
$\mathcal{O}_2^A =$ $\bar{q}\frac{1-\gamma_5}{2}\tau^a q\bar{q}\frac{1-\gamma_5}{2}\tau^a q$	$-\frac{1}{12}$	$\langle\mathcal{O}_2^A\rangle_0 - (3\langle\mathcal{O}_1^A\rangle_0 + \langle\mathcal{O}_2^A\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(-36 - 360\frac{T^2}{12F_\pi^2})$
$\mathcal{O}_2^B =$ $\bar{q}\frac{1-\gamma_5}{2}t^i q\bar{q}\frac{1-\gamma_5}{2}t^i q$	$\frac{3}{128}$	$\langle\mathcal{O}_1^B\rangle_0 - (3\langle\mathcal{O}_1^B\rangle_0 + \langle\mathcal{O}_2^B\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(-64 + 384\frac{T^2}{12F_\pi^2})$
$\mathcal{O}_2^B =$ $\bar{q}\frac{1-\gamma_5}{2}t^i\tau^a q\bar{q}\frac{1-\gamma_5}{2}t^i\tau^a q$	$-\frac{1}{128}$	$\langle\mathcal{O}_2^B\rangle_0 - (3\langle\mathcal{O}_1^B\rangle_0 + \langle\mathcal{O}_2^B\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(192 + 384\frac{T^2}{12F_\pi^2})$
$\mathcal{O}_1^C =$ $\bar{q}\frac{1-\gamma_5}{2}\sigma_{\mu\nu}t^i q\bar{q}\frac{1-\gamma_5}{2}\sigma_{\mu\nu}t^i q$	$-\frac{9}{512}$	$\langle\mathcal{O}_1^C\rangle_0 - (3\langle\mathcal{O}_1^C\rangle_0 + \langle\mathcal{O}_2^C\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(768 - 4608\frac{T^2}{12F_\pi^2})$
$\mathcal{O}_2^C =$ $\bar{q}\frac{1-\gamma_5}{2}\sigma_{\mu\nu}t^i\tau^a q\bar{q}\frac{1-\gamma_5}{2}\sigma_{\mu\nu}t^i\tau^a q$	$\frac{9}{1536}$	$\langle\mathcal{O}_2^C\rangle_0 - (3\langle\mathcal{O}_1^C\rangle_0 + \langle\mathcal{O}_2^C\rangle_0)\frac{T^2}{12F_\pi^2}$	$\frac{\langle\bar{u}u\rangle_0^2}{144}(2304 - 4608\frac{T^2}{12F_\pi^2})$

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